

Quantum Rings in Magnetic Fields and Spin Current Generation

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We propose three different mechanisms for pumping spin-polarized currents in a ballistic circuit using a time-dependent magnetic field acting on an asymmetrically connected quantum ring at half filling. The first mechanism works thanks to a rotating magnetic field and produces an alternating current with a partial spin polarization. The second mechanism works by rotating the ring in a constant field; like the former case, it produces an alternating charge current but the spin current is d.c.; both methods do not require a spin-orbit interaction to achieve the polarized current, but the rotating ring could be used to measure the spin-orbit interaction in the ring using characteristic oscillations. On the other hand, the last mechanism that we propose depends on the spin-orbit interaction in an essential way, and requires a time-dependent magnetic field in the plane of the ring. This arrangement can be designed to pump a purely spin current. The absence of a charge current is demonstrated analytically. Moreover, a simple formula for the current is derived and compared to the numerical results.

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I. INTRODUCTION AND BACKGROUND

Spintronics is an ambitious project which is still largely hypothetical but is making fast progress in connection with potential applications in memory devices, optoelectronics, and, among others, quantum information processing¹⁻⁴. The most basic question is how to generate the spin currents and inject them efficiently into the envisaged spintronic circuits. Johnson and Silsbee⁵ in a pioneering work excited a spin current from ferromagnetic electrodes into Al stripes. Now there are several other proposals. One is based on adiabatic modulations of magnetic stripes⁶; another adiabatic proposal is based on the competition between normal and Andreev reflections⁷. Early methods to (partially) polarize currents, like the Datta-Das transistor⁸, use a two-dimensional electron gas and an effective magnetic field due to the spin-orbit interaction. Indeed it can be seen e.g. in the review by Shen⁹ that most approaches for producing spin polarized currents are based on the spin-orbit interaction. The effectiveness of the process and the degree of polarization depends on the Rashba interaction, which has been measured at low temperatures in InAs, one of the promising materials¹⁰. Quite recently, Sadreev and Sherman explored the possibility of controlling the spin-flip conductivity in a wire; they found that this is feasible by crossed electric and magnetic fields exploiting the Rashba spin-orbit effect¹¹. It was shown that by modulating the transversal electric field one can produce some spin polarization in the current through the wire.

The spin current can be measured⁹. Remarkably, not only most routes for spin current generation but also detection methods are also based on the spin-orbit interac-

tion which can convert spin currents to charge currents¹².

Here we focus on the possible use of rings, which for fundamental topological reasons offer a complementary and also in part an alternative to the mechanism driven by the spin-orbit interaction to polarize the current and send it across a circuit. We wish to present several novel ways to produce and pump highly polarized spin currents based on the use of normal metal rings. Incidentally, a recent paper¹³ already deals with a one-dimensional quantum ring containing few electrons, endowed with a Rashba interaction, linear in the electron momentum. The result is that the Rashba effects, which is a part of the spin-orbit coupling, generates a pure spin current¹³; however the spin current is confined in the ring since there are no wires in the model.

We are interested in connected rings, instead. Besides producing the spin polarized current, one must solve the problem of efficiently injecting this current into the spintronic circuit. Classically the spin currents are of course unknown and at any rate there is no way to excite a current in a circuit without using a bias across its ends. In mesoscopic or ballistic conductors, however, quantum mechanics produces several kinds of pumping phenomena that have been highlighted by several authors.¹⁴⁻¹⁸.

A preliminary question naturally arises: *is it possible in principle to use rings to achieve pumping?* The answer is *yes* for nontrivial reasons rooted into the nonlinear quantum nature of the spin magnetic moment of nano-rings. This statement needs an immediate explanation and the reasoning runs as follows.

A planar connected ring has a magnetic moment which develops when a current is excited by a bias across the circuit, provided that the connection to the circuit breaks

parity in the plane. Classically the magnetic moment is linear in the current and in the bias¹⁹. Consider now the the ring in the absence of a bias. The effect of a magnetic flux in the ring is to excite a current in its sides. Provided the flux also pumps current in the external circuit, taking a net amount of charge from a wire to the other, then one has an example of one-parameter pumping. However the Brouwer theorem²⁰ forbids one-parameter pumping in a linear system. At this stage of the reasoning the answer to the above question is definitely *no*. This prediction is reversed by the finding²¹ that the magnetic moments excited by currents in nanoscopic circuits containing loops are dominated by quantum effects and depend nonlinearly on the exciting bias, quite at variance from classical expectations of a linear behavior.

Now we expect that ring can be used for pumping under suitable conditions and this has been verified²² by studying the real-time quantum evolution of tight-binding models in different geometries. It yields a nonadiabatic pumping. An arbitrary amount of charge can thereby be transferred from one side to the other by suitable flux protocols. Furthermore, in order to take electron-electron interactions into account, a quantum ring laterally connected to open one-dimensional leads was described within the Luttinger liquid model.²³ The interactions do not hamper the pumping effect. By a different arrangement one can model a memory device, in which both operations of writing and erasing can be performed efficiently and reversibly. As a direct consequence of the above mentioned nonlinearity, one can achieve, by employing suitable flux protocols, single-parameter nonadiabatic pumping.

The above findings hold for spinless models but here we wish to show that they open new directions in the search for practical methods to manipulate spins and spin currents. Thus, the present paper aims at extending the theory to include the spin- field and spin-orbit interactions and looking for new means for producing and pumping spin currents. We consider laterally connected normal metal rings with any even number of sides; the restriction to even numbers has a fundamental mathematical reason that will be apparent in Sect. VI below. Ring and leads are taken to be at half filling, (unlike the ring of Ref¹³), with no external bias.

The advantage of the present approach is that it starts from different topology, or genus; we can achieve high and even total spin polarization by three different methods that can work even at room temperature. The results depend in an essential way on the spin magnetic interaction, whereas one of them can also do without the spin-orbit coupling. The effects of the spin-orbit interactions on the above thought experiments are also explored and play a crucial role in the last arrangement. The conditions for a maximally spin polarized current are presented below in all cases. The spin polarizers that we propose are based on the use of a normal (i.e. neither superconducting nor magnetic) metal with half-filled bands.

The plan of the paper is as follows. Sect. II is devoted

to the geometry and formalism, Sect. III explains our research strategy. Sect. IV describes the spin current generated by a rotating magnetic field. In Sect. V it is the ring that rotates, keeping the spin quantization axis fixed. Sect. VI describes the pure spin current obtained by a variable field in the plane of the ring; the proof that the current is a pure spin current is given in Appendix. Sect. VII presents our conclusions.

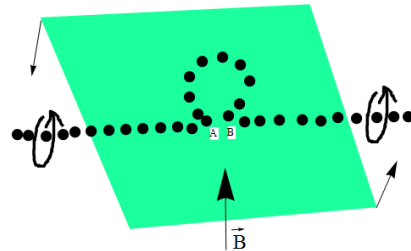


FIG. 1: We consider laterally connected rings, that are maximally asymmetric. In Sect.IV we consider the ring fixed in a rotating magnetic field; in Sect. V, we show that if the ring rotates around the AB bond in a fixed magnetic field, as depicted here, the time-dependent flux produces a spin-polarized current. In Sect.VI instead we study the effect of a tangential magnetic field.

II. LATERALLY CONNECTED RING IN MAGNETIC FIELD

The three thought experiments that we wish to perform use a ring shaped as a regular polygon with N_{ring} sides, laterally connected to wires, plunged in an external magnetic field. The geometry of the laterally connected ring is sketched in Fig.1 for $N_{ring} = 10$ and in Fig. 4a) for $N_{ring} = 8$. Since the effects that we introduce below are topological the precise value of a and the geometrical details are not crucial; taking for simplicity a regular polygon, the surface area is

$$S = \frac{N_{ring}a^2}{4 \tan(\pi/N_{ring})},$$

where the side length a is of the order of one Angstrom. In the ring the electrons feel the field and a spin-orbit interaction. There is no bias applied to the circuit; the time-dependent external magnetic field \vec{B} acting on the ring has a component $B_{||}$ in the plane of the ring and a normal component B_{\perp} which produces a flux $\alpha = B_{\perp}S$. For the sake of definiteness we assume that each bond in the ring is modified by the field according to the symmetric prescription $t_{ring} \rightarrow t_{ring} \exp[i\alpha(t)/N_{ring}]$, although the symmetry is not important here.

We outline the above situation by the tight-binding model Hamiltonian

$$H = H_D + H_B, \quad (1)$$

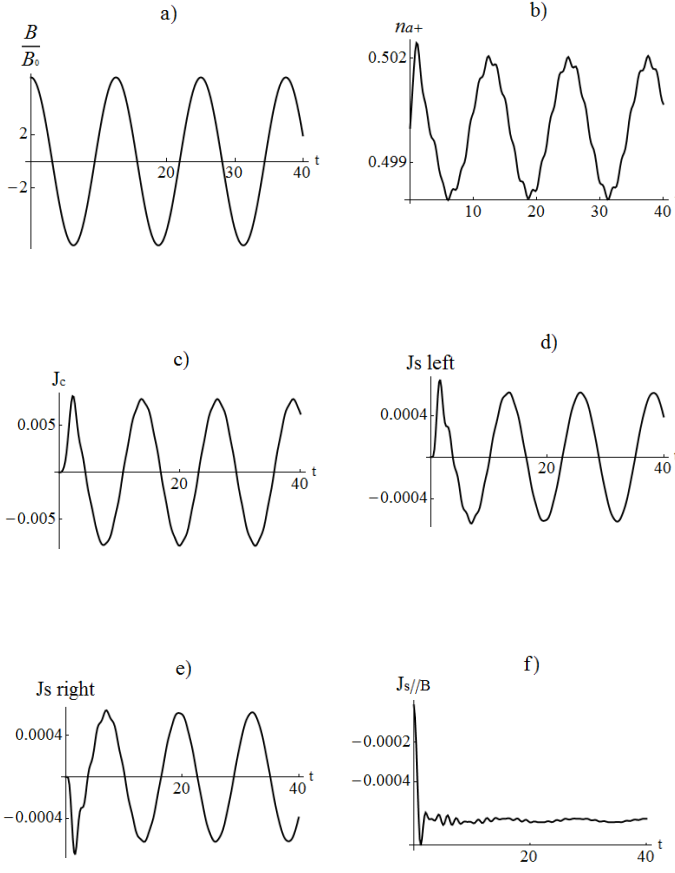


FIG. 2: Here we consider a $N_{ring} = 250$ atom ring without spin-orbit interaction. Time t is measured in $\frac{\hbar}{t_h}$ units. In Figures 2a) -2e) the ring is fixed in an external rotating magnetic field, with $\omega = \frac{2\pi t_h}{12.5\hbar}$. Figure 2a) shows normal component $B_L(t)$ of the magnetic field in units of $B_0 = \frac{\hbar c}{S_e}$, where $S = \frac{N_{ring} a^2}{4 \tan(\frac{\pi}{N_{ring}})}$ is the surface of the ring. So, B_L produces a fluxon in the ring. Fig. 2b): the spin up occupation of site A, where the ring is joined to the left lead. Figure 2c): charge current on the first bond of the left lead. Figure 2d): spin current on the first bond of the left lead, polarized normal to the ring. Figure 2d): spin current on the first bond of the left lead, polarized normal to the ring. Figure 2e): spin current on the first bond of the right lead, polarized normal to the ring; the spin current has a negative parity. In Figure 2f) the ring is rotating with angular frequency ω in a constant external magnetic field. and we show the spin current on the first bond of the left lead, polarized parallel to the magnetic field B .

where H_D describes the device and H_B the magnetic term.

$$H_D = H_{ring} + H_{wires} + H_{ring-wires}. \quad (2)$$

The N_{ring} -sided ring, with even N_{ring} , lies in the $x - y$ plane and is represented by

$$H_{ring} = t_{ring} e^{i \frac{\alpha(t)}{N_{ring}}} \sum_{\sigma=\pm 1} \exp[i\sigma\alpha_{SO}] c_{i+1,\sigma}^\dagger c_{i,\sigma} + h.c. \quad (3)$$

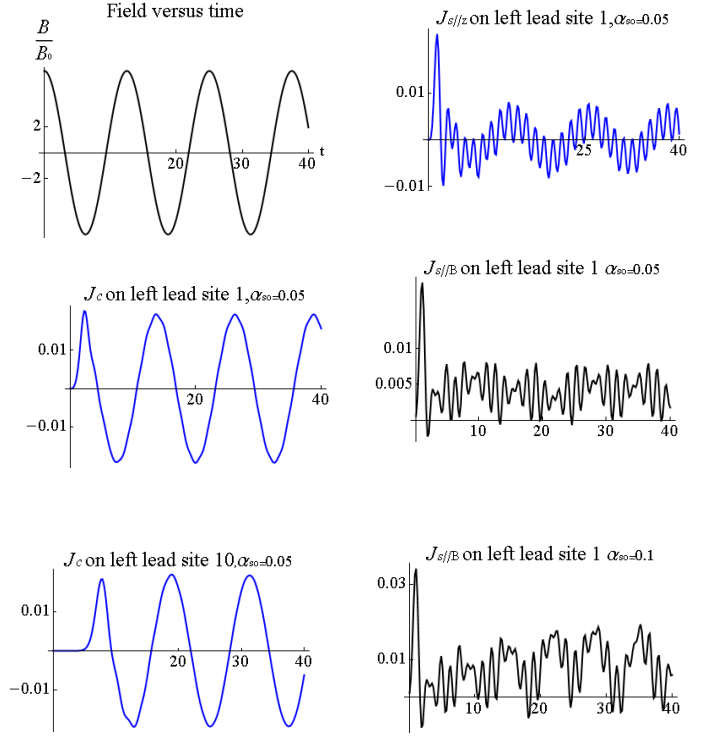


FIG. 3: Here we consider a 100 atom ring rotating in an external magnetic field with $\omega = \frac{2\pi t_h}{12.5\hbar}$. The spin-orbit interaction producing a phase shift ($\alpha_{SO} = 0.05$). Top left: the field as in Fig.2 versus time (in $\frac{\hbar}{t_h}$ units). Middle left: charge current on the first bond of the left lead. Bottom left: charge current on the tenth bond of the left lead. Top right: spin current on the first bond of the left lead, polarized normal to the ring. A modulation at frequency higher than ω is due to the spin-orbit interaction. Middle right: spin current on the first bond of the left lead, polarized parallel to the magnetic field B . Compared to Fig. 2e) the spin current is much larger, with wide oscillations. The difference is a size effect combined with an effect of the spin-orbit coupling. Bottom right: the same except that $\alpha_{SO} = 0.1$

where as usual $c_{i,\sigma}$ annihilates an electron at site i of spin projection $\sigma \frac{\hbar}{2}$, the quantization axis being the z axis. We also use the common pictorial notations $c_{i,\uparrow} = c_{i,+}$ and $c_{i,\downarrow} = c_{i,-}$. The further magnetic interaction due to $B_{||}$ acts exclusively on spin and is taken to be:

$$H_B = V(t) \sum_{i \in ring} (c_{i,\uparrow}^\dagger c_{i,\downarrow} + c_{i,\downarrow}^\dagger c_{i,\uparrow}) \quad (4)$$

where $V = \mu_B B$ with the Bohr magneton $\mu_B = 5.79375 * 10^{-5} \frac{eV}{Tesla}$.

The form of the spin-orbit interaction is borrowed from A. A. Zvyagin²⁴, where it is represented as a phase shift α_{SO} for up-spin and $-\alpha_{SO}$ for down-spin electrons. This implies that opposite spin electrons feel opposite effective fluxes in the ring. In this way the z direction becomes a privileged one in spin space.

In order to model the Left and Right wires, that extend, say, along the x axis, we introduce the correspond-

ing creation and annihilation operators and write:

$$H_{wires} = \sum_{\eta=Left,Right} H_{\eta}, \quad (5)$$

where

$$H_{\eta} = t_h \sum_{n,\sigma} c_{n,\sigma}^{\dagger(\eta)} c_{n+1,\sigma}^{(\eta)} + h.c., \quad (6)$$

The ring-wires contacts consist of hopping integrals t_{lr} that connect two nearest-neighbor sites of the ring denoted with A and B with the first sites of lead L and R, respectively.

In the Hamiltonian they are represented by a term

$$H_{leads-ring} = t_{lr} \left(\sum_{\sigma} c_{1,\sigma}^{\dagger Left} c_{A,\sigma} + \sum_{\sigma} c_{1,\sigma}^{\dagger Right} c_{B,\sigma} \right) + h.c.$$

Below we assume for the sake of definiteness that $t_h = t_{ring} = t_{lr} = 1$ eV.

The equilibrium occupation of the system (which prevails for $t < 0$) is determined by the spin-independent chemical potential μ_c . In this paper we consider $\mu_c = 0$ (i.e. we assume half filling).

In order to describe its evolution of the system starting from its ground state, we begin our consideration from the retarded Green's function matrix elements in a spin-orbital basis:

$$g_{i,j}^r = \langle i | U_I(t, 0) | j \rangle \quad (7)$$

where $U_I(t, 0)$ is the evolution operator in the interaction representation. Taking the spin quantization axis along z , the number current²⁵ is

$$J_{n,\sigma}(t) = -2 \frac{t_h}{\hbar} \text{Im}(G_{n,\sigma,n-1,\sigma}^<) \quad (8)$$

where

$$G_{i,j}^<(t) = \sum_{\mu} n_{\mu}^0 g_{i,\mu}^r(t, 0) g_{j,\mu}^{r*}(t, 0). \quad (9)$$

where μ denotes the ground state spin-orbitals for $B = 0$ and n_{μ}^0 is the Fermi function. The tight-binding model is a simple but often useful approximation; one advantage is that it allows exact results, as those concerning the pure spin current derived in the Appendix.

A. Units used in the numerical calculations

We shall simulate several experimental settings. In all cases, our codes calculate number currents taking the hopping integral $t_h = 1$. For instance if this is interpreted to mean that $t_h = 1$ eV, which corresponds to the frequency $2.42 * 10^{14} \text{ s}^{-1}$, a current $J = 1$ from the code means $2.42 * 10^{14}$ electrons per second, which corresponds to a charge current of $3.87 * 10^{-5}$ Ampere.

III. THE PHYSICAL PROBLEM: SPIN-DEPENDENT PUMPING

We may start our reasoning from the study of the spinless case²² where adding a flux quantum to the flux piercing the ring is a gauge transformation. While the Hamiltonian is left invariant, the transient electric field pumps a finite amount of charge towards the left or the right wire according to the sign of the flux. The spin magnetic moment spoils the invariance of the Hamiltonian producing an energy shift between up and down spins (quantized normal to the ring plane), and any pumping mechanism will now be spin-polarized. Since the field needed to produce a fluxon is inversely proportional to the surface, the spinless model²² continues to work approximately for large enough rings while it breaks down for small ones. Below we consider three possibilities to overcome this; the first two are based on the idea that in a cyclic process up and down spins are again put on equal footing, while the last works with a field in the ring plane, which again treats up and down spins (quantized normal to the ring plane) on equal footing. Finally, in the present analysis we take the bands to be half filled, for the sake of having all the possible symmetry, and the ring is taken with an even number of sides, since in this way the whole system is bipartite. This is particularly important in Sect. 6.

IV. ROTATING MAGNETIC FIELD

In the first arrangement that we consider in this work we leave the ring fixed in the x - y plane while the field rotates; in Cartesian coordinates, $\vec{B}(t) = (0, B_{\parallel}(t), B_{\perp}(t))$, with $B_{\parallel} = B \cos(\omega t)$ and $B_{\perp} = B \sin(\omega t)$. In Figures 2a) to 2d) we report the results of a simulation with a 250 atom ring and $\omega = \frac{2\pi t_h}{12.5\hbar}$ in which $\alpha_{SO} = 0$, i.e. no spin-orbit interaction is included. The magnetic flux has a sine dependence on time shown in Figure 2a). In the numerical code the wires are 50 atoms long, which is enough to simulate infinite wires for the time interval up to $40 \frac{\hbar}{t_h}$ that we are considering, since longer wires give the same results.

Fig. 2b) shows the time dependence of the spin up occupation of site A, where the ring is joined to the left lead. The charge is seen to oscillate without showing a trend to deviate steadily from half filling. In Figure 2c) we see a similar oscillatory trend of the charge current on the first bond of the left lead.

Figures 2d) and 2e) show the spin current on the first sites of the left and right lead, respectively; the spin quantization axis is in the z direction orthogonal to the ring. As above, $J = 1$ from the code means $2.42 * 10^{14}$ spins per second, if $t_h = 1$ eV, and the spin current is proportional to t_h .

We conclude that a rotating magnetic field around a fixed ring would pump an alternating spin current in the wires. The driving force can be understood in terms of

the contents of the previous Section. At this stage, the results are already striking: an oscillating charge current is to be expected even classically in the ring, but the pumping is a purely quantum mechanical phenomenon. Once the sense of rotation of the magnetic field is chosen, clockwise and anticlockwise are made physically different and up and down spins have different energies in the adiabatic limit; it is clear that this situation produces the alternating charge current. Moreover, one can see that changing the sense of rotations all currents must change sign, as the numerical solution shows. In addition, this reasoning suggests that the B dependence must be essentially linear and that the spin-orbit interaction does not play an essential role. However, pumping has already been described in a spin-less model²². The most important novelty is the spin current, which is obtained without the help of the spin-orbit interaction, which has always been used to polarize currents in previous works, as mentioned above. The new ingredient is the topology, which *per se* allows to produce spin currents. An even more striking possibility comes next.

V. ROTATING RING

The second arrangement that we consider in this work is shown in Figure 1, where the ring rotates in a constant field. In the reference of the ring the field still rotates according to $B_{||} = B \cos(\omega t)$ and $B_{\perp} = B \sin(\omega t)$, as in the previous case. The difference is that now the spin quantization axis is kept parallel to the magnetic field B . The current $J_m(\vec{n})$ at site m along $\vec{n} = (\sin(\theta), 0, \cos(\theta))$ reads:

$$J_m(\vec{n}) = \cos^2\left(\frac{\theta}{2}\right)J_{m,+} + \sin^2\left(\frac{\theta}{2}\right)J_{m,-} + \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)J_{m,+}^{sf} \quad (10)$$

where we introduced the spin-flip current at site m

$$J_m^{sf} = \sum_{\sigma} (c_{m+1,\sigma}^{\dagger} c_{m,-\sigma} - c_{m,\sigma}^{\dagger} c_{m+1,-\sigma}). \quad (11)$$

Consequently, we are interested in the the spin current polarized along $\theta = \omega t$:

$$J_m^{spin}(\vec{n}) = J_m(\vec{n}) - J_m(-\vec{n}) \quad (12)$$

given in terms of Equation (7) by

$$J_m^{spin}(\vec{n}) = (J_{m,+} - J_{m,-}) \cos(\theta) + J_m^{sf} \sin(\theta). \quad (13)$$

In Figure 2e) we see the result. The ring pumps in the wires an oscillating charge current but a *d.c.* spin current. In this example the magnitude of the field is taken to be such that at normal incidence the ring is pierced by a fluxon ($\frac{h}{e} = 4.134 * 10^{-15}$ MKSA units). This means that for a 100 atom ring the magnetic field must be of the order of $\frac{500}{a^2}$ Tesla, where a is in Angstroms. This is admittedly too strong a field for the current technology, however the field decreases

with increasing N_{ring} and, at any rate, the currents are strictly linear with B . In fact, the arguments of the previous Section concerning the linearity and the secondary role of the spin-orbit interaction remain valid here.

In Figure 3, a 100 atom ring with spin-orbit interaction ($\alpha_{SO} = 0.05$) rotates in a constant external magnetic field. The top left frame shows the magnetic flux versus time (in $\frac{h}{t_h}$ units). Below we report the computed charge current on the first bond of the left lead. The current is measured in electrons per unit time, with the time unit $\frac{h}{t_h}$ appropriate for this model. The current oscillates with the same periodicity as the phase, as one could expect in linear response. In the bottom left position of Fig.3 we see the same current, but referred to the 10th site; there is an evident delay because the disturbance takes time to get there, but then the pattern is the same.

In Figure top right we show the spin current on the first bond of the left lead, keeping the spin quantization axis normal to the rotating ring. It is about 25% of the charge current and follows roughly the same oscillatory pattern. In Figure 3 middle right we show the spin current on the first bond of the left lead, but now with the spin quantization axis parallel to the magnetic field B . The spin current on the first bond of the right lead is exactly opposite to this. There is an evident change of pattern between Figure 2e) and Figure 3d) which is essentially due to the spin-orbit interaction. Finally, Figure 3 bottom right is like the previous one but with $\alpha_{SO} = 0.1$. The intensity of the spin current increases and the oscillations are wider.

VI. MAGNETIC FIELD IN THE PLANE OF THE RING: PURE SPIN CURRENT

The next thought experiment that we propose to produce a spin current requires a time-dependent magnetic field in the plane of the ring. So, $B_{\perp} = 0$ and while there is no magnetic flux, $B_{||} = B(t)$ acts on the spin degrees of freedom. The ring is taken in the x-y plane and the spin-polarized current is excited by a time-dependent external magnetic field $B(t)$ along the x axis.

Here the assumption that N_{ring} is even is crucial. The analysis of the time evolution is enormously simplified and can be carried out with generality for any $V(t)$ since the model is bipartite (i.e. bonds connect sites of two disjoint sublattices), and can be mapped on a spin-less model which is also bipartite (Fig. 4). A Dirac monopole in the middle (the star in Fig. 4b)) is the point source of a magnetic field which pierces both sub-clusters symmetrically; however it enters the upper ring from below and the lower ring from above. In this way, imparting opposite chirality to the two sub-clusters, it ensures opposite phase shifts in the corresponding bonds, and this represents the spin-orbit interaction (see Eq. 3). Due to the spin-orbit interaction the parity $P : x \rightarrow -x$ and the

reflection Σ which sends each sub-cluster to the other fail to commute with H , but the product $P\Sigma$ is a symmetry.

Let us consider sites at the same distance from the ring on both leads and use the correspondence $\alpha \rightarrow$ left, spin up; $\beta \rightarrow$ left, spin down; $\gamma \rightarrow$ right, spin up, and $\delta \rightarrow$ right, spin down. Then, the $P\Sigma$ symmetry implies that at any time, the currents J and the charge densities ρ are constrained by

$$\begin{aligned} J_\alpha(t) &= -J_\delta(t) \\ J_\beta(t) &= -J_\gamma(t) \\ \rho_\alpha(t) &= \rho_\delta(t) \\ \rho_\beta(t) &= \rho_\gamma(t). \end{aligned} \quad (14)$$

In general, the charge on the ring will fluctuate. The question arises: what is the condition for the ring to stay neutral? Introducing the 'and' symbol \bigwedge , the continuity equation requires:

$$J_\alpha(t) = J_\beta(t) \bigwedge J_\gamma(t) = J_\delta(t). \quad (15)$$

Combining with the conditions arising from the $P\Sigma$ symmetry, one finds

$$J_\alpha(t) = -J_\gamma(t) \bigwedge J_\beta(t) = -J_\delta(t), \quad (16)$$

which means, there should be a pure spin current. In general these conditions cannot be met and the ring gets charged, but at half filling these symmetries combined with the charge conjugation symmetry imply that the charge current vanishes identically. This is shown in the Appendix.

A. How the spin current arises

A vanishing current would trivially satisfy the above symmetries and theorem in the Appendix. Instead, the numerical integration of the Schrödinger equation reveals that a spin current flows. Since this is a novel effect, we must investigate what is the driving force for the spin current and how one can predict analytically its magnitude. The perturbation treatment in the small parameter $\frac{V}{t_h}$ although elementary, produces very large formulas which are of no use here. In order to achieve a simple estimate of the effect, and capture the essential mechanism producing the spin current, we need to introduce the concept that the pumping action by a ring on the outside circuit can be represented by an effective bond. We shall start with the spinless case and then extend the treatment to the present problem.

1. Effective bond

As a preliminary, in order to motivate the renormalised bond idea, let us consider the simpler problem of spinless electrons in the same device, but with a normal magnetic

field producing a flux in the ring. Such a model was studied previously²²; it was shown that by suitable protocols one can insert an integer number of fluxons in the ring in such a way that the electronic system in the ring is not left charged and is not excited, while charge is pumped in the external circuit.

Writing the number current in units of $\frac{t_h}{h}$ it turns out that $\int J dt$ is of order unity for every fluxon. In this case, the ring is equivalent to an effective bond with hopping $t_h \rightarrow t_h \exp(i\beta(t))$. The time dependent vector potential entails the effective bias across the bond is $e\phi_{eff} = \hbar\dot{\beta}$. The quantum conductivity of the wire was discussed elsewhere²⁵; at small ϕ_{eff} , the number current is $J = -\frac{\phi_{eff}}{\pi\hbar}$. Integrating over time, one finds that the total charge pumped when a fluxon is swallowed by the ring is $Q = \int J dt = \frac{\beta}{\pi} \sim 1$. In other terms, inserting a flux quantum in the ring we shift an electron in the characteristic hopping time of the system. This simple argument is in good agreement with the numerical results²².

2. The driving force

For the present purposes, we now show that we can replace the ring by a renormalised bond, with hopping $t_h \rightarrow \tau \exp(i\beta_\sigma(t))$ with $\tau \sim t_h$, which implies an effective potential drop across the bond which produces the current. Indeed, in terms of the Peierls prescription, this implies a spin-dependent electric field \vec{E}_σ such that $\dot{\beta}_\sigma = \frac{2\pi}{h} \int e \vec{E}_\sigma d\vec{l}$. The phase and the effective potential are spin-dependent and produce the spin current.

In the case with spin, the above approach leads us to change the equivalent model of Figure 4b) to the simplified model of Figure 5, where the vertical bonds again stand for $V(t)$ and the effective bond bears a spin-orbit induced static phase $\alpha \sim \alpha_{SO}$ which produces no effect at all for $V=0$. When $V(t)$ is on, however, the electron wave function in the upper wire can interfere with a time dependent contribution from the opposite spin sector where the phase shift is opposite. Effectively this works like a time dependent phase drop across the upper bond, and an opposite phase drop across the lower one. The spin-dependent electric field is the driving force producing the effect.

We are now in position to estimate the magnitude of the spin current. In first-order perturbation theory the amplitude to go from k_{1+} in the upper wire to k_{2-} in the lower wire reads

$$c_\alpha(k_1, k_2, t) = \frac{-i}{\hbar} \int_0^t d\tau e^{i\omega(k_2, k_1)\tau} [1 + e^{-2i\alpha + i\Delta_{12}}] V(\tau) \quad (17)$$

where $\omega(k_2, k_1) = 2(\cos(k_2) - \cos(k_1))$ and $\Delta_{12} = k_1 - k_2$. Since the graph of Fig. 5 is also bipartite, the current J is spin-dependent and site-independent, and $J\psi_k = \frac{2t_h}{h}\psi_k$. Therefore the mean current on the top wire is obtained

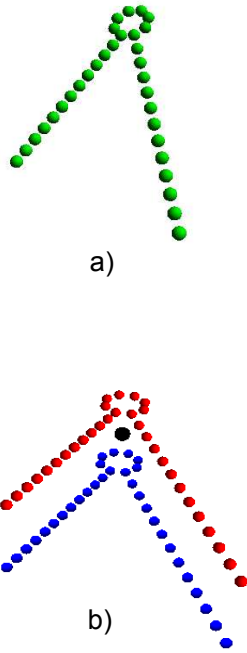


FIG. 4: Maximally asymmetric connection of the $N=6$ ring to wires. The circles with up and down arrows represents the sites; those in the hexagon feel the spin-orbit interaction and the magnetic field. a) Geometry of the device and the magnetic field; all sites are connected horizontally to the first neighbors by spin-diagonal matrix elements, and B flips spins in the hexagon. b) Equivalent cluster for spinless electrons. The star represents the Dirac monopole providing the effective spin-orbit interaction. All sites are connected horizontally to the first neighbors, and those in the ring have also vertical bonds due to the V magnetic interactions.

by summing over occupied states:

$$\langle J_\alpha \rangle = \frac{2t_h}{\hbar} \sum_{\kappa_1}^{\cos(k_1) < 0} \sum_{\kappa_2}^{\cos(k_2) > 0} \sin(k_2) |c_\alpha(k_1, k_2, t)|^2. \quad (18)$$

By definition, the spin current is $J_s(\alpha) = J_\alpha - J_{-\alpha}$. As a simple example, let us take

$$V(t) = V\theta(t)\theta(T-t). \quad (19)$$

Then, by taking with $V = \mu_B B$ and then letting $T = \frac{\hbar}{t_h}$ (short rectangular spike) we obtain

$$J_s(\alpha) \sim \frac{t_h}{\hbar} \frac{\sin(\alpha)}{2\pi} \left(\frac{VT}{\hbar} \right)^2. \quad (20)$$

This result is in good agreement with the magnitude of the computed spin currents (see below). We conclude that the effective bond concept allows for a simple qualitative picture of the effect.

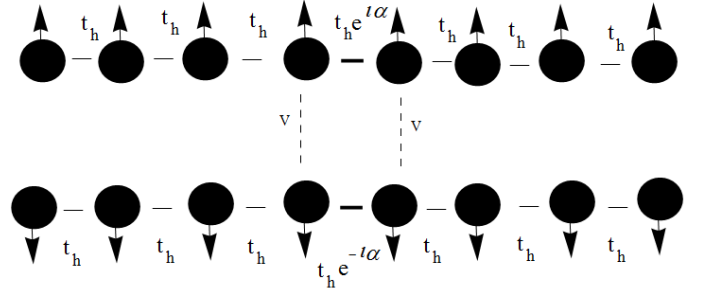


FIG. 5: The simplified version of our model used to derive Eq. (14). The top (bottom) circles represent the up (down) spin states of a chain; in both horizontal lines the continuous lines stand for identical real hopping matrix elements t_h while the dotted lines represent $t_h e^{i\alpha}$ (top) and $t_h e^{-i\alpha}$ (bottom) connecting, say, sites 0 and 1. The vertical lines stand for $V(t)$ time-dependent hoppings that replace the magnetic interactions in the simplified model.

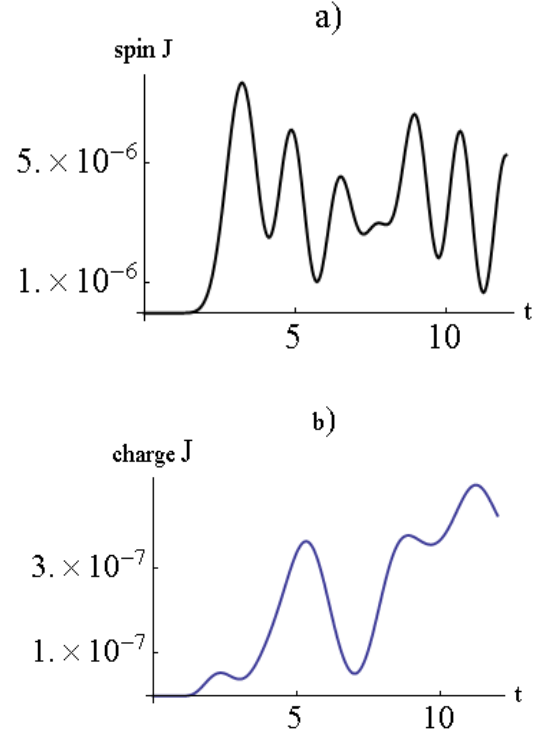


FIG. 6: Numerical results of the model of Eq. (1) with 100 sites in the ring and 100 sites in each lead. The currents are excited by the sudden switching of a field $B=100$ Tesla and is quadratic with B . Units are given in the text. Panel a): spin current J_s at half filling and zero temperature. The results hardly change if one takes a temperature of $0.025 t_h$, of the order of room temperature if $t_h \sim 1\text{eV}$. In both cases the charge current vanishes exactly. Panel b): assuming zero temperature and a Fermi energy $0.01 t_h$, the spin current does not change, but a charge current does not vanish any more, although it is still an order of magnitude smaller than the spin current.

B. Field in the plane of the ring: Numerical results

Test calculations were computed for the full model according to Equation (7). As in the previous Sections, the wires are represented in the codes by chains of N_{wires} atoms; if N_{wires} is so large that the results do not change by increasing N_{wires} we consider that $N_{wires} \rightarrow \infty$. The actual N_{wires} which is necessary in a given calculation increases with the time duration that one wants to represent. $G^<$ was obtained from the determinantal wave function; the code evolves the quantum state by a time-slicing integration of the Schrödinger equation.

The numerical results strikingly illustrate the above analytic findings. In particular, for any time dependence of $V(t)$ the charge current vanishes identically at half filling.

In Fig. 6 we present the results²⁶ for the case of a sudden switching of B . We also tested the validity of the simple approximation of Eq.20 for the full model. For $B = 100$ Tesla and $\alpha = 1$ one finds $J_s = 5 * 10^{-6} \frac{t\hbar}{h}$. The numerical response to a narrow delta-like spike yields $J_s = 6 * 10^{-6} \frac{t\hbar}{h}$ and the quadratic dependence on B is fully confirmed. So the simple approximation works for the full model and although $B = 100$ Tesla is high one can easily scale the result to laboratory fields.

Finite temperatures do not change significantly the results up to $K_B T \sim 0.025$ eV. This is interesting since up to now, strongly spin-polarized currents have been created and detected in ultra-cold atomic gases only²⁷. Instead, the results are sensitive to the filling. In the lower panel of Figure 6 one can see the result of raising the Fermi level to $E_F = 0.01$. A relatively small charge current develops while numerically the spin current appears to be unaffected by the shift of E_F . As shown at the start of Sect. VI this implies that the ring gets charged in the process.

Then, we move to show the numerical results²⁸ of our model for a linear time-dependence of B (Fig 7a)) in a numerical experiment on a hexagonal ring; the infinite wires are simulated with 45 sites long chains.

Conventionally we take the hopping matrix elements equal to 1 eV and time units are chosen accordingly. In Fig. 7b) we report the spin current at the first site of each wire (the two are equal since as already remarked spin current is even). The roughly parabolic time dependence is in line with our conclusion that the field dependence is *grosso modo* quadratic in this case (while in the rotating ring experiment above it was linear). In 7c) and 7e) we report the computed spin-up and spin-down currents in the first site of the left wire, while in 7d) and 7f) we show the same currents for the right wire. This illustrates the positive parity of both currents and the exact vanishing of the charge current.

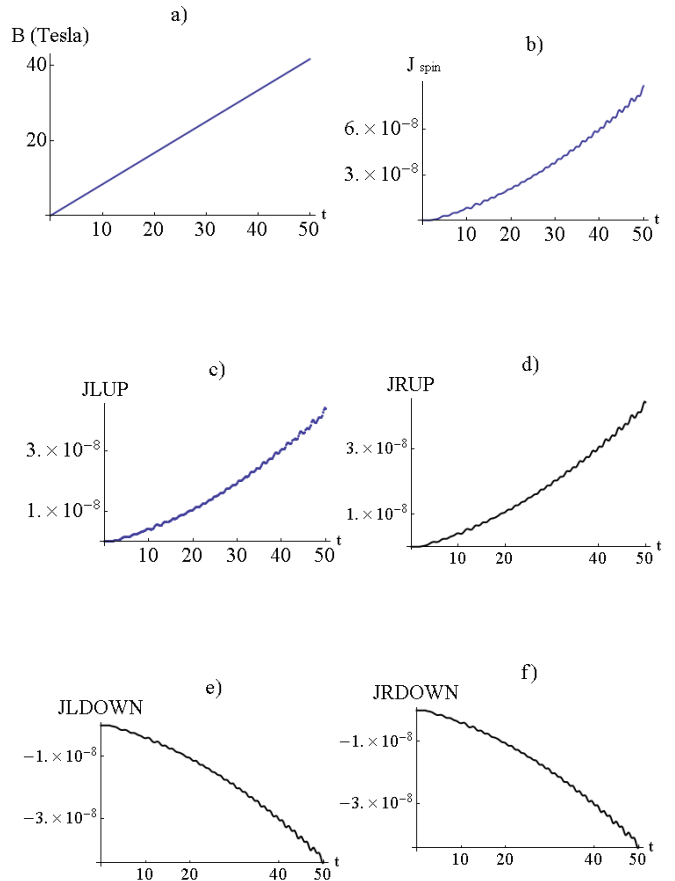


FIG. 7: Results of a numerical experiment with an hexagonal ring and 45 sites long wires. a) Time dependence of the in-plane magnetic field; b) spin current on the first site of both wires. To explain the spin-space distribution of the currents, the computed up spin currents on the first site of left and right wires are reported in c) and d) respectively; they are the same. The spin-down currents in the same sites are reported in (e) and right wire (f) and are the negative of the up-spin currents. This shows that the spin current is even and the charge current vanishes identically.

In Fig. 8 we use the same model system but with a sinusoidal time dependence of $B(t)$, shown in Fig. 8a). In Fig. 8b) we report the spin-up current response on the 8th site of the left wire; it is identical to the spin-up current on the symmetric site of the right wire and opposite to the spin-down current on both sides. So, even in this case, a pure spin current is excited. Interestingly, however, the response of figure 8b) is not exactly a sine, and a doubled frequency prevails.

In the above numerical simulations we have illustrated different ways to pump a pure spin current into a ballistic circuit. The mechanisms advocated in this work do not require magnetic materials but rely upon either the rotating of the ring in an external magnetic field in the absence of a spin-orbit interaction, or, as an alternative, upon the combined effects of the spin-orbit interaction, and a peculiar choice of the geometry that

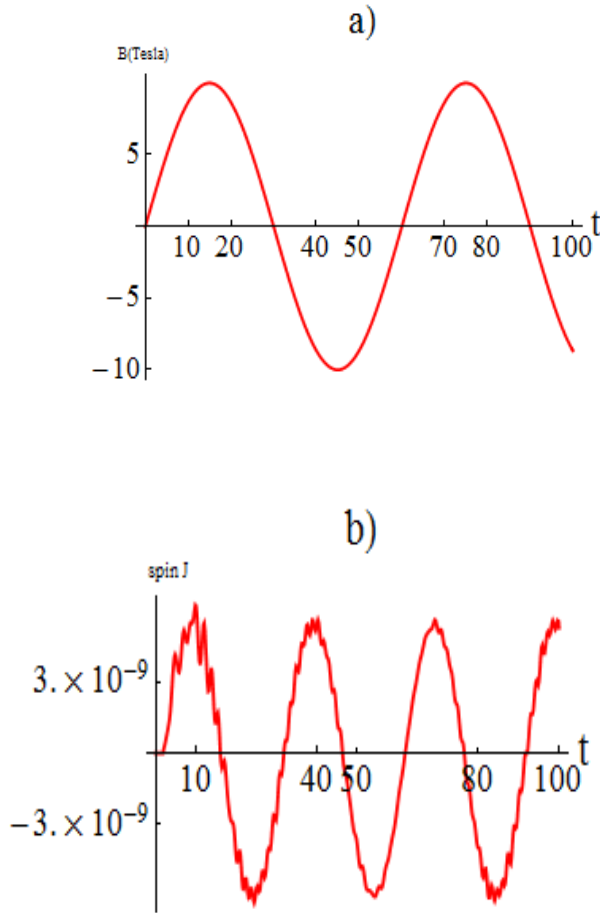


FIG. 8: a) Sinusoidal magnetic field, with period $60\frac{\hbar}{t_h}$, used in the second numerical experiment ;b) spin current response on the 8th site of the left wire.

exploits the maximally asymmetric ring and the external magnetic field.

VII. CONCLUSION AND OUTLOOK

We have shown that in principle by using asymmetrically connected rings at half filling and a time-dependent magnetic field one can pump spin-polarized currents in a ballistic circuit. Actually there are at least three novel different mechanisms available, that after a proper engineering could be of interest for practical applications.

A rotating magnetic field piercing the ring pumps an a.c. spin current with partial spin polarization and does not rely on the spin-orbit coupling.

Alternatively one can obtain a d.c. spin current by letting the ring rotate in a static field; the spin-orbit coupling modulates the time dependence but is not required for the spin polarization. The current is proportional to the field.

The third mechanism achieves the pumping of a purely

spin current thanks to both the spin-orbit interaction, and the action of a tangent time-dependent magnetic field. The time dependence of the spin current follows from the time dependence of the field. The current is approximately quadratic in the field in this case.

Ultimately, all these results are consequences of purely quantum effects first discussed in Ref.²¹, since the classical theory that one finds on Jackson's book¹⁹ would not allow any sort of pumping.

The main message of the present investigation is that by a proper use of quantum rings and magnetic fields one can master the technical problems of spin current handling in future spintronics without the need for nothing else than normal metals. Heavy metals with a large spin-orbit coupling can be needed for some applications involving the last of the three methods proposed here.

VIII. APPENDIX: PROOF OF THE PURE SPIN CURRENT

The above findings can be put on a rigorous basis for the sake of the mathematically oriented reader. The study is simplest in the equivalent lattice of Fig. 4b). Let us consider first any eigenstate of the instantaneous H , thought of as stationary. Changing sign to all the amplitudes in a sublattice, we get a solution of the one-electron problem with opposite hopping t_h , and also a solution of the same Schrödinger equation with opposite energy. Hence, if ϵ is an energy eigenvalue, $-\epsilon$ also is, and opposite energy eigenstates must have the same probability on site. Coming to the many-body state, the sum of the probabilities is exactly one half. In other terms, each site of the equivalent lattice is exactly half filled, and in the original model the two spin states are exactly half filled on every site. This holds for any static B including the initial state where $B = 0$.

In the adiabatic limit the system is in the instantaneous ground state at each time. Then, no charge current is allowed, because the total occupation of each site in Fig. 4a) is fixed; moreover no spin current is allowed either, since it would alter the occupation of the sites in Fig. 4b), which is also bipartite. Therefore the adiabatic evolution of this system is trivial. Since we are interested in the non-adiabatic evolution, the beautiful analysis by Avron et al.²⁹ does not apply here.

Next, we consider the time evolution in the presence of the time-dependent field. To show that during the time evolution $B(t)$ produces a pure spin current in the half filled system, we change to a staggered spin-reversed hole representation with $c_{i,\sigma}^\dagger = s b_{i,-\sigma}$, where $s = 1$ in one sublattice and $s = -1$ in the other; here the operators refer to any site of the device. The transformation of any bond goes as follows: $t_{n,m}^\sigma c_{n,\sigma}^\dagger c_{m,\sigma} \rightarrow -t_{n,m}^\sigma b_{n,-\sigma} b_{m,-\sigma}^\dagger$. Since t^σ is Hermitean this is the same as $t_{m,n}^{\sigma*} b_{m,-\sigma}^\dagger b_{n,-\sigma}$ and since opposite spins have conjugate hoppings we may

rewrite this as $t_{m,n}^{-\sigma} b_{n,-\sigma}^\dagger b_{n,-\sigma}$. On the other hand, in the ring the sites coupled by V belong to opposite sublattices and so the transformation gives:

$$H_B \rightarrow V(t) \sum_{i \in \text{ring}} (b_{i,\uparrow}^\dagger b_{i,\downarrow} + b_{i,\downarrow}^\dagger b_{i,\uparrow}). \quad (21)$$

In this way, at every time t the transformed hole Hamiltonian $\tilde{H}(b, b^\dagger)$ depends on b operators exactly as the original Hamiltonian $H(c, c^\dagger)$ depends on the c operators. In both pictures the evolution starts in the ground state at half filling and evolves in the same way. Therefore, at any time and for any site n ,

$$\langle b_{n,\sigma}^\dagger(t) b_{n,\sigma}(t) \rangle_b = \langle c_{n,\sigma}^\dagger(t) c_{n,\sigma}(t) \rangle_c. \quad (22)$$

Here the l.h.s. is the average at time t in the b picture while the r.h.s. is averaged at time t in the c picture. Hence, operating the canonical transformation on the l.h.s.,

$$\langle 1 - n_{n,-\sigma}(t) \rangle_c = \langle n_{n,\sigma}(t) \rangle_c, \quad (23)$$

which implies that the mean total occupation of each site is conserved. This cannot be true if charge currents exist. Indeed, let us consider the operators straddling each bond: since at each time

$$\langle b_{n+1,\sigma}^\dagger b_{n,\sigma} \rangle_b = \langle c_{n+1,\sigma}^\dagger c_{n,\sigma} \rangle_b \quad (24)$$

we may conclude that

$$\langle c_{n,-\sigma}^\dagger c_{n+1,-\sigma} \rangle_c = \langle c_{n+1,\sigma}^\dagger c_{n,\sigma} \rangle_c. \quad (25)$$

Hence the current is pure spin current, q.e.d.

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